A Formalism for Weak Interactions in Nuclei

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The neutrino-nucleus cross-section and the muon capture rate are discussed within a simple formalism which facilitates the nuclear structure calculations. The corresponding formulae only depend on four types of nuclear matrix elements, which are currently used in the nuclear beta decay. We have also considered the non-locality effects arising from the velocity-dependent terms in the hadronic current. We show that for both observables in ^{12}C the higher order relativistic corrections are of the order of $\sim$ 4% only, and therefore do not play a significant role.

The semileptonic weak interactions with nuclei include a rich variety of processes, such as the neutrino (antineutrino) scattering, charged lepton capture, $e^{\pm}$ decays, etc, and we have at our disposal the results of more than a half-century of beautiful experimental and theoretical work. At the time being their study is mainly aimed to inquire on possible exotic properties of the neutrino associated with its oscillations and massiveness by means of exclusive and inclusive scattering processes on nuclei, which are often used as neutrino detectors. An example is given by the recent experiments performed by both the LSND and the KARMEN Collaborations, looking for $\nu_\mu \to \nu_e$ and $\tilde{\nu}_\mu \to \tilde{\nu}_e$ oscillations with neutrinos produced by accelerators [1, 2]. Thus the knowledge of reactions induced by neutrinos on nuclei becomes a crucial step for the interpretation of experiments on neutrinos, and a reliable prediction of these cross sections is a challenging problem from the nuclear structure point of view.

The weak interaction formalism most frequently used in the literature is that of Walecka [3], where, in close analogy with the electromagnetic transitions, the nuclear operators are classified into Coulomb ($\mathcal{M}$), longitudinal ($\mathcal{L}$), transverse electric ($\mathcal{T}^{el}$) and transverse magnetic ($\mathcal{T}^{mag}$). We carry out a different multipole expansion of the $V-A$ hadronic current and express all observables in terms of the vector (V) and axial vector (A) nuclear matrix elements.

The weak Hamiltonian is expressed in the form [4]

$$H_W(\mathbf{r}) = \frac{G}{\sqrt{2}} J^\dagger_\mu L^\mu(\mathbf{r}) + h.c., \tag{1}$$

where

$$J_\mu = \gamma_0 \left[g_{\rm V}\gamma_\mu + \frac{g_{\rm M}}{2M} i\sigma_{\mu\nu}k_\nu - g_{\rm A}\gamma_\mu\gamma_5 + \frac{g_{\rm P}}{m_\ell} k_\mu \gamma_5 \right], \tag{2}$$

is the hadronic current operator, and

$$L_\mu(\mathbf{r}) = \overline{u}_{s_\ell}(\mathbf{p}, E_\ell)\gamma_\mu(1-\gamma_5)u_{s_\nu}(\mathbf{q}, E_\nu)e^{-i\mathbf{r}\cdot\mathbf{k}} \tag{3}$$

is the plane waves approximation for the matrix element of the leptonic current; $G = (3.04545 \pm 0.00006)\times 10^{-12}$ is the Fermi coupling constant (in natural units),

$$k = P_i - P_f \equiv \{k_0, \mathbf{k}\} \tag{4}$$

is the momentum transfer (P_i and P_f are momenta of the initial and final nucleon (nucleus), M is the nucleon mass, m_ℓ is the mass of the charged lepton, and g_V, g_A, g_M and g_P are, respectively, the vector, axial-vector, weak-magnetism and pseudoscalar effective dimensionless coupling constants. Their numerical values are

$$\begin{aligned} g_V &= 1;\ g_A = 1.26; \\ g_M &= \kappa_p - \kappa_n = 3.70;\ g_P = g_A \frac{2Mm_\ell}{k^2 + m_\pi^2}. \end{aligned} \tag{5}$$

The finite nucleon size (FNS) effect is incorporated via the dipole form factor with a cutoff $\Lambda = 850$ MeV, *i.e.,* as:

$$g \to g\left(\frac{\Lambda^2}{\Lambda^2 + k^2}\right)^2.$$

To use (1) with the non-relativistic nuclear wave functions, the Foldy-Wouthuysen transformation has to be performed on the hadronic current (2). When the velocity dependent terms are included this yields [1] :

$$\begin{aligned} J_0 &= g_{\rm V} + (\overline{g}_{\rm A} + \overline{g}_{\rm P1})\boldsymbol{\sigma}\cdot\hat{\mathbf{k}} - \frac{g_{\rm A}}{M}\boldsymbol{\sigma}\cdot\mathbf{p} \\ \mathbf{J} &= -g_{\rm A}\boldsymbol{\sigma} - i\overline{g}_{\rm W}\boldsymbol{\sigma}\times\hat{\mathbf{k}} - \overline{g}_{\rm V}\hat{\mathbf{k}} + \overline{g}_{\rm P2}(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})\hat{\mathbf{k}} + \frac{g_{\rm V}}{M}\mathbf{p}, \end{aligned} \tag{6}$$

[1] There is a misprint in eq.(5) of Ref.[5].

where the operator $\mathbf{p} \equiv -i\boldsymbol{\nabla}$ acts on the nuclear wave functions, and the following short notation has been introduced:

$$\begin{aligned} \overline{g}_{\rm V} &= g_{\rm V}\frac{|\mathbf{k}|}{2M};\ \overline{g}_{\rm A} = g_{\rm A}\frac{|\mathbf{k}|}{2M};\ \overline{g}_{\rm W} = (g_{\rm V}+g_{\rm M})\frac{|\mathbf{k}|}{2M}, \\ \overline{g}_{\rm P1} &= g_{\rm P}\frac{|\mathbf{k}|}{2M}\frac{k_0}{m_\ell};\ \overline{g}_{\rm P2} = g_{\rm P}\frac{|\mathbf{k}|}{2M}\frac{|\mathbf{k}|}{m_\ell}. \end{aligned} \tag{7}$$

For the neutrino-nucleus reaction $k = p_\ell - q_\nu$, with $p_\ell \equiv \{E_\ell, \mathbf{p}_\ell\}$ and $q_\nu \equiv \{E_\nu, \mathbf{q}_\nu\}$, and the corresponding cross section from the initial state $|J_i\rangle$ to the final state $|J_f\rangle$ reads

$$\sigma(E_\ell, J_f) = \frac{|\mathbf{p}_\ell|E_\ell}{2\pi}F(Z+1,E_\ell)\int_{-1}^{1} d(\cos\theta)\mathcal{T}_\sigma(|\mathbf{k}|,J_f), \tag{8}$$

where $F(Z+1,E_\ell)$ is the Fermi function, $\theta \equiv \hat{\mathbf{q}}_\nu\cdot\hat{\mathbf{p}}_\ell$, and

$$\mathcal{T}_\sigma(|\mathbf{k}|,J_f) \equiv \frac{1}{2J_i+1}\sum_{s_\ell,s_\nu,M_i,M_f}\left|\int d\mathbf{r}\psi_f^*(\mathbf{r})H_W(\mathbf{r})\psi_i(\mathbf{r})\right|^2,$$

with $\psi_i(\mathbf{r}) \equiv \langle\mathbf{r}|J_iM_i\rangle$ and $\psi_f(\mathbf{r}) \equiv \langle\mathbf{r}|J_fM_f\rangle$ being the nuclear wave functions.

The transition amplitude can be cast in the form:

$$\begin{aligned} \mathcal{T}_\sigma(|\mathbf{k}|,J_f) = G^2\sum_J \Big\{ & L_4\left[g_V^2|\mathcal{M}_V(J)|^2 + |(\overline{g}_A+\overline{g}_{P1})\mathcal{M}_A^0(J) + g_A\mathcal{M}_{A'}(J)|^2\right] \\ & + L_0\left[\Re\left[\left(\overline{g}_{\rm V}\mathcal{M}_V(J) + 2g_{\rm V}\mathcal{M}^0_{V'}(J)\right)\overline{g}_{\rm V}\mathcal{M}_V^*(J)\right] + \left(\overline{g}_{\rm P2}^2 - 2g_{\rm A}\overline{g}_{\rm P2}\right)|\mathcal{M}_A^0(J)|^2\right] \\ & + \sum_{\mu=0,\pm1} L_\mu\left|(g_{\rm A}-\mu\overline{g}_{\rm W})\mathcal{M}_A^\mu(J) - g_{\rm V}\mathcal{M}^\mu_{V'}(J)\right|^2 \\ & + 2L_{40}\Re\Big[g_{\rm V}\left(\overline{g}_{\rm V}\mathcal{M}_V(J) + g_V\mathcal{M}^0_{V'}(J)\right)\mathcal{M}_V^*(J) \\ & + (g_{\rm A}-\overline{g}_{\rm P2})\left((\overline{g}_{\rm A}+\overline{g}_{\rm P1})\mathcal{M}_A^0(J) + g_{\rm A}\mathcal{M}_{A'}(J)\right)\mathcal{M}_A^{0*}(J)\Big]\Big\}, \end{aligned} \tag{9}$$

where the lepton traces L_4, L_{40} and $L_{\mu=-1,0,1}$ are defined in Ref.[5] , and the nuclear matrix elements

$$\begin{aligned} \mathcal{M}_V(J) &= \frac{(-1)^J}{\sqrt{2J_i+1}}\langle J_f||\mathsf{Y}_J(|\mathbf{k}|)||J_i\rangle, \\ \mathcal{M}_A^\mu(J) &= \frac{1}{\sqrt{2J_i+1}}\sum_L\sqrt{2L+1}\begin{pmatrix} L & 1 & J \\ 0 & -\mu & \mu\end{pmatrix} \times\langle J_f||\mathsf{S}_{JL}(|\mathbf{k}|,\boldsymbol{\sigma})||J_i\rangle, \\ \mathcal{M}_{A'}(J) &= \frac{(-1)^J}{\sqrt{2J_i+1}}\langle J_f||\mathsf{Y}_J(|\mathbf{k}|,\boldsymbol{\sigma}\cdot\mathbf{p})||J_i\rangle, \\ \mathcal{M}^\mu_{V'}(J) &= \frac{1}{\sqrt{2J_i+1}}\sum_L\sqrt{2L+1}\begin{pmatrix} L & 1 & J \\ 0 & -\mu & \mu\end{pmatrix} \times\langle J_f||\mathsf{P}_{JL}(|\mathbf{k}|,\mathbf{p})||J_i\rangle, \end{aligned} \tag{10}$$

contain the operators :

$$\begin{aligned} \mathsf{Y}_{JM}(|\mathbf{k}|) &= \sqrt{4\pi}i^J j_J(|\mathbf{k}|r)Y_{JM}(\hat{\mathbf{r}}) \\ \mathsf{S}_{JLM}(|\mathbf{k}|,\boldsymbol{\sigma}) &= \sqrt{4\pi}j_L(|\mathbf{k}|r)i^L\left[Y_L(\hat{\mathbf{r}})\otimes\boldsymbol{\sigma}\right]_{JM} \\ \mathsf{P}_{LJM}(|\mathbf{k}|,\mathbf{p}) &= \frac{1}{M}\sqrt{4\pi}i^L j_L(|\mathbf{k}|r)[Y_L(\hat{\mathbf{r}})\otimes\mathbf{p}]_{JM}, \\ \mathsf{Y}_{JM}(|\mathbf{k}|,\boldsymbol{\sigma}\cdot\mathbf{p}) &= \frac{1}{M}\sqrt{4\pi}i^J j_J(|\mathbf{k}|r)Y_{JM}(\hat{\mathbf{r}})(\boldsymbol{\sigma}\cdot\mathbf{p}). \end{aligned} \tag{11}$$

Similarly, for the capture rate one gets [5]

$$\Lambda(J_f) = \frac{E_\nu^2}{2\pi}|\phi_{1S}|^2\mathcal{T}_\Lambda(J_f), \tag{12}$$

where ϕ_{1S} is the muonic bound state wave function evaluated at the origin, and

$$\begin{aligned} \mathcal{T}_\Lambda(J_f) = G^2\sum_J\Big\{ & \left|G_V\mathcal{M}_V(J) + g_{\rm V}\mathcal{M}^0_{V'}(J)\right|^2 \\ & + \left|G_{A0}\mathcal{M}_A^0(J) - g_{\rm A}\mathcal{M}_{A'}(J)\right|^2 \\ & + 2\left|G_{A1}\mathcal{M}_A^1(J) - g_{\rm V}\mathcal{M}^1_{V'}(J)\right|^2\Big\}, \end{aligned} \tag{13}$$

where the effective charges are

$$\begin{aligned} G_{A0} &= -g_{\rm A} - \overline{g}_{\rm A} + \overline{g}_{\rm P}, \qquad G_V = g_{\rm V} + \overline{g}_{\rm V}, \\ G_{A1} &= -g_{\rm A} - \overline{g}_{\rm W}. \end{aligned} \tag{14}$$

and the coupling constants are now

$$\begin{aligned} \overline{g}_{\rm V} &= g_{\rm V}\frac{E_\nu}{2M};\ \overline{g}_{\rm A} = g_{\rm A}\frac{E_\nu}{2M}; \\ \overline{g}_{\rm W} &= (g_{\rm V}+g_{\rm M})\frac{E_\nu}{2M};\ \overline{g}_{\rm P} = \overline{g}_{\rm P2} - \overline{g}_{\rm P1} = g_{\rm P}\frac{E_\nu}{2M}. \end{aligned}$$

As an application, we have evaluate the contribution of the non-locality effects, arising from the velocity-dependent terms in the hadronic current, in weak processes with neutrinos and muons within the triad $\{^{12}B, ^{12}C, ^{12}N\}$. The numerical calculations were performed within the particle number projection charge-exchange RPA (PQRPA) [6], by employing the same configuration space ($nl = (1s, 1p, 1d, 2s, 1f, 2p)$) and the same residual force ($V = -4\pi\left(v_sP_s + v_tP_t\right)\delta(r)$) as in Ref.[5]. We also use the same parameterization as in this work for the coupling strengths within the particle-particle (pp) and particle-hole (ph) channels: $v_s^{ph} = v_s^{pair} = 23.92$ MeV-fm^3, and $v_t^{ph} = v_s^{ph}/0.6 = 39.86$ MeV-fm^3.

In the Table I are given the results for inclusive folded flux-averaged neutrino scattering cross sections

$$\langle\sigma_\ell(J_f)\rangle = \int dE_\nu\sigma(E_\ell = E_i - E_f - E_\nu, J_f)\bar{f}(E_\nu), \tag{15}$$

with $\ell = e^-, \mu^-$, and where $\bar{f}(E_\nu)$ is the normalized neutrino flux.

We have presented a somewhat new formalism for the neutrino-nucleus cross-section and the muon capture rate, based on a multipole expansion of the $V-A$ hadronic current in terms of the transition operators (13), which are currently used in the beta decay. The resulting formulae are more simpler than those developed by Walecka [3] and as such facilitate the nuclear structure calculations. We have also considered the non-locality effects arising from the velocity-dependent terms in the hadronic current. We show that for both observables in ^{12}C the higher order relativistic corrections are of the order of $\sim 4\%$ only, and therefore do not play a very significant role.

TABLE 1. Partial contributions to the inclusive folded flux-averaged cross section $\nu_e(^{12}C,^{12}N)e^-$ $(10^{-42}\ cm^2)$, inclusive $\nu_\mu(^{12}C,^{12}N)\mu^-$ $(10^{-40}\ cm^2)$ and inclusive muon capture ($10^3\ s^{-1}$). In parenthesis are showed the results obtained in Ref. [5], which do not include the velocity dependent terms.

Processes	PQRPA (I)
$\nu_e(^{12}C,^{12}N)e^-$	21.7 (21.6)
$\nu_\mu(^{12}C,^{12}N)\mu^-$	14.7 (14.6)
$\mu^-(^{12}B,^{12}C)\nu_\mu$	48.1 (46.0)

References

[1] LSND Collaboration, C. Athanassopulus *et al.* , Phys. Rev. C**58**, 2489 (1998).

[2] KARMEN Collaboration, K. Eitel *et al.* , Nucl. Phys. **B**, Proc.Suppl. **77**, 212 (1999).

[3] J.D. Walecka, *Muon Physics*, ed. V.M. Hughes and C.S Wu (Academis, New York, 1975).

[4] C. Barbero, F. Krmpotić, and D. Tadić, Nucl. Phys. A**628**, 170 (1998); C. Barbero, F. Krmpotić, A. Mariano and D. Tadić, Nucl. Phys. A**650**, 485 (1999).

[5] F. Krmpotić, A. Mariano and A. Samana, Phys.Lett. B**541**, 298 (2002).

[6] F. Krmpotić, A. Mariano, T.T.S. Kuo, and K. Nakayama, Phys. Lett. B**319**, 393 (1993).